SNUTP 01-004, hep-ph/0101170

Lepton Flavor Violation and Radiative Neutrino Masses

Eung Jin Chun

Department of Physics, Seoul National University, Seoul 151-747, Korea

Abstract

Lepton flavor violation in various sectors of the theory can bring important effects on neutrino masses and mixing through wave function renormalization. We examine general conditions for flavor structure of radiative corrections producing the atmospheric and solar neutrino mass splittings from degenerate mass patterns. Also obtained are the mixing angle relations consistent with the experimental results.

Current data coming from the atmospheric [1] and solar neutrino experiments [2] strongly indicate oscillations among three active neutrinos following one of the mass patterns: (i) hierarchical pattern with $|m_1|, |m_2| \ll |m_3|$, (ii) inversely hierarchical pattern with $|m_1| \simeq |m_2| \gg |m_3|$, (iii) almost degenerate pattern with $|m_1| \simeq |m_2| \simeq |m_3|$. In each case, one needs the mass-squared differences $\Delta m^2_{atm} = \Delta m^2_{32} \simeq \Delta m^2_{31} \sim 3 \times 10^{-3}$ eV^2 and $\Delta m^2_{sol} = \Delta m^2_{21} \sim 10^{-4} - 10^{-10}$ eV^2 for the atmospheric and solar neutrino oscillations, respectively [3]. Here we define $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ for the neutrino mass eigenvalues m_i. For the mixing angles, we take the standard parameterization of the neutrino mixing matrix U,

$$U = R_{23}(\theta_1)R_{13}(\theta_2)R_{12}(\theta_3) = \begin{pmatrix} c_2c_3 & s_3c_2 & s_2 \\ -c_1s_3 - s_1s_2c_3 & c_1c_3 - s_1s_2s_3 & s_1c_2 \\ s_1s_3 - c_1s_2c_3 & -s_1c_3 - c_1s_2s_3 & c_1c_2 \end{pmatrix}, \tag{1}$$

where $R_{ij}(\theta_k)$ is the rotation in the ij plane by the angle $0 \le \theta_k \le \pi$ and $c_k = \cos\theta_k$, $s_k = \sin\theta_k$. In our discussion, we neglect CP-violating phases. The atmospheric neutrino data require the $\nu_\mu - \nu_\tau$ oscillation amplitude $A_{atm} = c_2^2 \sin^2 2\theta_1 \approx 1$ implying nearly maximal mixing $\theta_3 \approx \pi/4$. For the solar neutrino oscillation, the $\nu_e - \nu_{\mu,\tau}$ oscillation amplitude $A_{sol} = c_2^2 \sin^2 2\theta_3$ can take either large ~ 1 or small $\sim 10^{-3}$ value (that is, $\theta_1 \sim \pi/4$ or $\theta_1 \ll 1$) depending on the solutions to the solar neutrino problem. The mixing element U_{e3} is constrained by the reactor experiment on the $\bar{\nu}_e$ disappearance [4]; $|U_{e3}| = |s_2| \lesssim 0.2$ for Δm^2_{32} allowed by the atmospheric neutrino data. There is also a limit on the neutrino mass element itself coming from the neutrinoless double beta decay experiments, $|M_{ee}| = |\sum_i m_i U^2_{ei}| < 0.2$ eV [5]. This bound is particularly important for the degenerate pattern (iii) where $m^2_i \gg \delta m^2_{atm}$ is possible. Note that we restricted ourselves to oscillations among three active neutrinos disregarding the LSND results [6] waiting for a confirmation in the near future.

Understanding the origin of neutrino masses and mixing is one of fundamental problems in physics beyond the standard model. It concerns with the leptonic flavor structure of the theory. As we know, small Majorana neutrino mass textures (at tree level) would be attributed to the effective higher dimensional operator $L_\alpha L_\beta H_2 H_2$ where L_α denotes the lepton doublet with the flavor index $\alpha = e, \mu, \tau$ and H_2 is the Higgs field coupling to up-type quarks. This operator breaks the total lepton number by 2 units ($\Delta L = 2$). There can be other sectors breaking individual lepton number (with $\Delta L = 0$) which give important contributions to the neutrino mass matrix through radiative corrections. The best-known example is the renormalization group effect of the charged lepton Yukawa couplings [7], which has been studied extensively in recent years [8]. Such an effect provides potentially important origin of tiny neutrino mass splittings, of course, depending on specific tree level neutrino mass textures.

In this regard, it is an interesting question whether the desired mass splittings for the degenerate mass patterns (ii) and (iii) can arise from radiative corrections bringing the effect of lepton flavor violation in various sectors of the theory. The purpose of this work is to investigate general conditions for radiative corrections to produce the observed neutrino mass splittings without resorting to any specific models for flavor structure beyond the Yukawa sector. Assuming that various radiative correction contributions are hierarchical, we will identify appropriate flavor structure of the dominant or subdominant contribution which are able to generate the atmospheric or solar neutrino mass splitting. For these loop corrections, we also derive mixing angle relations consistent with the experimental data. Our results would be useful for constructing models of degenerate neutrinos.

The general form of the loop-corrected neutrino mass matrix due to wave function renormalization is given by

$$M_{\alpha\beta} = m_{\alpha\beta} + \frac{1}{2}\left(m_{\alpha\gamma} I_{\gamma\beta} + I_{\alpha\gamma} m_{\gamma\beta}\right) , \tag{2}$$

where $m_{\alpha\beta}$ is the tree level neutrino mass matrix and $I_{\alpha\beta}$ is the loop contribution. Note that the above formula is written in the flavor basis where charged lepton masses are diagonal. Often, it is convenient to re-express Eq. (2) in the tree level mass basis of neutrinos;

$$M_{ij} = m_i \delta_{ij} + \frac{1}{2}(m_i + m_j) I_{ij} , \tag{3}$$

where m_i is the tree level mass eigenvalue and the loop correction factor I_{ij} is related to $I_{\alpha\beta}$ by the equation, $I_{ij} = \sum_{\alpha\beta} I_{\alpha\beta} U_{\alpha i} U_{\beta j}$, where U is the tree level diagonalization matrix parametrized as in Eq. (1). In this paper, we have nothing to mention about the origin of the flavor structure of $m_{\alpha\beta}$. Given $m_{\alpha\beta}$ yielding the degenerate mass pattern of the type (ii) or (iii), we will examine the flavor structure of the loop factor $I_{\alpha\beta}$ which can produce the atmospheric and/or solar neutrino masses and mixing. Before coming to our main point, it is instructive to see where the loop correction $I_{\alpha\beta}$ can come from.

To illustrate how the flavor structure of the loop correction $I_{\alpha\beta}$ can arise, we consider one of the most popular model beyond the standard model, namely, the supersymmetric standard model. Like the standard model, it has an inevitable flavor violation in the Yukawa sector with the superpotential,

$$W \ni h_\alpha H_1 L_\alpha E^c_\alpha , \tag{4}$$

where H_1, L_α and E^c_α denote the Higgs, lepton doublet and singlet superfields. At the leading log approximation, the Yukawa terms in Eq. (4) give rise to [7, 8]

$$I_{\alpha\alpha} \approx -\frac{h_\alpha^2}{8\pi^2} \ln\left(\frac{M_X}{M_Z}\right) , \tag{5}$$

where M_X denote a fundamental scale generating the above-mentioned effective operator and the Z boson mass M_Z represents the weak scale.

The soft supersymmetry breaking terms included in this model are also potentially important sources of flavor violation. Those terms include sfermion masses and trilinear A-terms which are generically nonuniversal and flavor dependent;

$$V_{soft} \ni m^2_{\alpha\beta^*} L_\alpha L^\dagger_\beta + A_{\alpha\beta} H_1 L_\alpha E^c_\beta + h.c. , \tag{6}$$

where we use the same notation for the scalar components of superfields. The effect of slepton masses has been first discussed in Ref. [11]. The off-diagonality and non-degeneracy in diagonal masses of $\tilde{m}^2_{\alpha\beta^*}$ give rise to

$$I_{\alpha\beta} = \frac{g^2}{8\pi^2} \delta^l_{\alpha\beta} f(\tilde{m}^2_{\alpha\alpha}; \tilde{m}^2_{\beta\beta}) , \tag{7}$$

where $\delta^l_{\alpha\beta} = \tilde{m}^2_{\alpha\beta}/(\tilde{m}_{\alpha\alpha}\tilde{m}_{\beta\beta})$ for $\alpha \neq \beta$ (assuming $\delta^l_{\alpha\beta} \ll 1$), $\delta^l_{\alpha\alpha} = 1$ and f is an appropriate loop function of order one. In the similar way, the effect of the A-terms comes from one-loop diagrams with wino/zino and slepton exchange generating

$$I_{\alpha\beta} \approx \frac{g^2}{8\pi^2} \frac{A_{\alpha\gamma} A^*_{\beta\gamma} \langle H_1 \rangle^2}{\tilde{m}^4} , \tag{8}$$

where $\tilde{m}$ is a typical mass of the sparticles running inside the loop. In general, the A-terms are not proportional to the (charged lepton) Yukawa couplings, $A_{\alpha\beta} \not\propto h_{\alpha\beta} = \delta_{\alpha\beta} h_\alpha$, This can lead to important new contributions to lepton flavor changing loop corrections as above. However, we expect in generic models that $A_{\alpha\beta} \lesssim \tilde{m} h_\tau$ and thus the sizes of their loop effects are at most comparable to those of the tau Yukawa coupling.

Lepton flavor violation can also appear in the superpotential through R-parity and lepton number violating trilinear terms;

$$W \ni \lambda_{\alpha\beta\gamma} L_\alpha L_\beta E^c_\gamma + \lambda'_{\alpha\beta\gamma} L_\alpha Q_\beta D^c_\gamma \,, \tag{9}$$

where Q, D^c are doublet and down-type singlet quark superfields. The R-parity violating couplings can participate in the renormalization group equation like the charge lepton Yukawa couplings, and thus we get for the couplings λ,

$$I_{\alpha\beta} \approx -\frac{\lambda_{\alpha\gamma\delta}\lambda^*_{\beta\gamma\delta}}{8\pi^2} \ln(\frac{M_X}{M_Z}) \,. \tag{10}$$

For certain combinations of R-parity violating couplings [9], the current experimental bounds are weak enough to give a sizable loop correction $I_{\alpha\beta}$. In particular, if one takes only one dominant coupling $\lambda_{\alpha\gamma\delta}$ which can be as large as order one, one can have a very large loop correction $I_{\alpha\alpha}$. With the generic R-parity violation (9), of course, one may have a finite loop correction to neutrino masses like $\delta m_{\alpha\beta} \propto \lambda_{\alpha\gamma\delta}\lambda_{\beta\delta\gamma} h^e_\gamma h^e_\delta$ [10] which is not a topic of the present investigation.

As we have seen above, there could be rich sources for sizable loop corrections $I_{\alpha\beta}$ in a general class of models. In the below, we discuss the effect of general flavor-dependent loop contributions $I_{\alpha\beta}$ for various degenerate mass patterns at tree level. Let us start with analyzing the conditions to produce the desired mass splittings for the atmospheric *as well as* solar neutrino oscillations in the case of the fully degenerate mass patterns; $|m_1| = |m_2| = |m_3|$. For this, we look first for the possible loop corrections giving rise to the atmospheric mass splitting $\Delta m^2_{32}/2m^2_0$ at the leading order correction. In our analysis, it is assumed that loop corrections $I_{\alpha\beta}$ take hierarchical values and thus the leading order splitting is dominated by one specific $I_{\alpha\beta}$. For the fully degenerate pattern, there are the following possibilities depending on the CP-conserving phases of the tree level mass eigenvalues;

$$\text{I.} \quad (m_1, m_2, m_3) = m_0(-1, -1, 1) \quad \text{or} \quad m_0(\pm 1, \mp 1, 1)$$

which we call the 1-2 or 1-3 (2-3) degeneracy, respectively. The loop correction can induce nonvanishing off-diagonal components M_{ij} as far as $m_i + m_j \neq 0$ (3). Then, as discussed in Ref. [11], one has a freedom to define the tree level mixing angles in the matrix U in such a way that $I_{ij} = I_{\alpha\beta} U_{\alpha i} U_{\beta j} = 0$ due to the exact degeneracy $m_i = m_j$. It is a simple manner to show this explicitly. In the i-j plane with tree-level degeneracy, the radiatively corrected mass matrix is diagonalized by the rotation $R_{ij}(\phi)$ where ϕ is given by

$$\tan 2\phi = \frac{2I_{ij}}{I_{jj} - I_{ii}} \,. \tag{11}$$

On the other hand, we are free to choose our tree level mixing matrix: $U \to U' = U R_{ij}(\phi)$ where $R_{ij}(\phi)$ is given by $R_{ij,kl}(\phi) = c_\phi(\delta_{ik}\delta_{il} + \delta_{jk}\delta_{jl}) + s_\phi(\delta_{ik}\delta_{jl} - \delta_{jk}\delta_{il})$. Then the loop factor $I'_{ij} = I_{\alpha\beta} U'_{\alpha i} U'_{\beta j}$ written in terms of the new mixing matrix U' becomes $I'_{ij} = [2I_{ij} \cos 2\phi +$

$(I_{ii} - I_{jj}) \sin 2\phi]/2$ which vanishes due to the relation (11). Note that this conclusion holds for arbitrary sum of $I_{\alpha\beta}$.

With this *properly* defined mixing matrix U satisfying the *mixing angle relation* $I_{ij} = 0$, the nonvanishing mass-squared differences generated by loop correction can be written as

$$\begin{aligned} \Delta m^2_{21} &= 2m_0^2(I_{22} - I_{11}) = 2m_0^2 I_{\alpha\beta}(U_{\alpha2}U_{\beta2} - U_{\alpha1}U_{\beta1}) \\ \Delta m^2_{32} &= 2m_0^2(I_{33} - I_{22}) = 2m_0^2 I_{\alpha\beta}(U_{\alpha3}U_{\beta3} - U_{\alpha2}U_{\beta2})\,. \end{aligned} \tag{12}$$

When a specific component $I_{\alpha\beta}$ gives the dominant contribution in Eq. (12), we need to have $(U_{\alpha2}U_{\beta2} - U_{\alpha1}U_{\beta1})/(U_{\alpha3}U_{\beta3} - U_{\alpha2}U_{\beta2}) \lesssim \Delta m^2_{sol}/\Delta m^2_{atm}$, that is, $U_{\alpha2}U_{\beta2} \approx U_{\alpha1}U_{\beta1}$ is required to get $|m^2_{32}| \gg |m^2_{21}|$. Based on these properties, we are now ready to consider the effect of a loop correction $I_{\alpha\beta}$.

(a) $I_{\alpha\alpha}$ dominance: For the i-j degeneracy, the relation $I_{ij} \propto U_{\alpha i}U_{\alpha j} = 0$ implies $U_{\alpha i} = 0$ or $U_{\alpha j} = 0$. As an immediate consequence, we find that the 1-2 degeneracy cannot work for any α since it gives $U^2_{\alpha1} \simeq U^2_{\alpha2} \simeq 0$ and thus $U^2_{\alpha3} \simeq 1$ contradicting with the empirical results, $U^2_{e3} \ll 1$ and $U^2_{\mu3} \simeq U^2_{\tau3} \simeq 1/2$. In fact, the only possible mixing angle relation is $U_{e3} = s_2 = 0$, which can be realized only with the I_{ee} dominance combined with the 1-3 or 2-3 degeneracy. In this case, we get from Eq. (12)

$$\frac{\Delta m^2_{21}}{\Delta m^2_{32}} \simeq \frac{\cos 2\theta_3}{s_3^2} \tag{13}$$

which requires the maximal mixing of the solar neutrinos, $\cos 2\theta_3 \ll 1$.

(b) $I_{\alpha\beta}$ dominance ($\alpha \neq \beta$): As discussed below Eq. (12), it is useful to notice that we need $0 \neq U_{\alpha2}U_{\beta2} \simeq U_{\alpha1}U_{\beta1} \simeq -U_{\alpha3}U_{\beta3}/2$ (the last relation comes from the orthogonality condition $\sum_i U_{\alpha i}U_{\beta i} = \delta_{\alpha\beta}$) to yield $\Delta m^2_{21}/\Delta m^2_{32} \approx 2(I_{22} - I_{11})/3I_{22}$. Then, we can easily rule out the case $(\alpha\beta) = (e\mu)$ or $(e\tau)$ from the simple observation that

$$\frac{\Delta m^2_{21}}{\Delta m^2_{32}} \simeq \frac{2}{3}\left[\cos 2\theta_3 + \frac{c_1}{s_1 s_2}\sin 2\theta_3\right] \quad \text{or} \quad \frac{2}{3}\left[\cos 2\theta_3 - \frac{s_1}{c_1 s_2}\sin 2\theta_3\right]$$

which cannot be made small enough for realistic values of the mixing angles satisfying $\theta_1 \simeq \pi/4$, $\theta_2 \ll 1$ and $\theta_3 \simeq \pi/4$ or $\theta_3 \ll 1$. On the other hand, for $(\alpha\beta) = (\mu\tau)$, we get

$$\frac{\Delta m^2_{21}}{\Delta m^2_{32}} \simeq -\frac{2}{3c_2^2}\left[(1 + s_2^2)\cos 2\theta_3 + 2s_2 \sin 2\theta_3 \cot 2\theta_1\right] \tag{14}$$

which becomes vanishingly small for $s_2 \cos 2\theta_1 \ll 1$ and $\cos 2\theta_3 \ll 1$. Now we have to check if the mixing angle relation fixed by the condition $I_{ij} = 0$ for certain combination of (ij) can be consistent with our consideration. As shown in Ref. [11], the 1-3 or 2-3 degeneracy gives again the desired relation respectively,

$$s_2 = -\cot 2\theta_1 \tan\theta_3 \quad \text{or} \quad s_2 = \cot 2\theta_1 \cot\theta_3\,. \tag{15}$$

which relates the smallness of the angle θ_2 with the large atmospheric neutrino mixing.

In sum, the small mass splitting for the atmospheric neutrinos can be obtained *only* with the dominant loop correction of I_{ee} or $I_{\mu\tau}$ in the case of the 1-3 or 2-3 degeneracy. Furthermore, this picture can be consistent only with *bimaximal* mixing $\theta_1, \theta_3 \simeq \pi/4$ and $s_2 \ll 1$ fixed by the mixing angle relation I_{13} or $I_{23} = 0$. Note that all of these are fairly

r	$\Delta m^2_{32}/m_0^2$	$\Delta m^2_{21}/m_0^2$	s_2
$I_{ee}/I_{\mu\tau}$		0	0
$I_{\mu\mu}/I_{\mu\tau}$		$I_{\mu\tau} r^2 \sin 2\theta_1/2$	$-rs_3/2c_3$
$I_{\tau\tau}/I_{\mu\tau}$	$I_{\mu\tau}\sin 2\theta_1$	$I_{\mu\tau} r^2 \sin 2\theta_1/2$	$rs_3/2c_3$
$I_{e\mu}/I_{\mu\tau}$		$I_{\mu\tau} r c_1 c_2 \sin 2\theta_3$	$r/2c_1c_2$
$I_{e\tau}/I_{\mu\tau}$		$-I_{\mu\tau} r s_1 c_2 \sin 2\theta_3$	$r/2s_1c_2$
$I_{\mu\mu}/I_{ee}$		$-I_{ee} r^2/2$	$rc_1s_1s_3/c_3$
$I_{\tau\tau}/I_{ee}$		$-I_{ee} r^2/2$	$-rc_1s_1s_3/c_3$
$I_{e\mu}/I_{ee}$	$-I_{ee}$	$I_{ee} r c_1 c_2 \sin 2\theta_3$	$-rs_1/c_2$
$I_{e\tau}/I_{ee}$		$-I_{ee} r s_1 c_2 \sin 2\theta_3$	$-rc_1/c_2$
$I_{\mu\tau}/I_{ee}$		0	0

TABLE I: Possibilities of radiative generation of Δm^2_{atm} and Δm^2_{sol} in the case of the degeneracy, $m_1 = -m_2 = m_3$. The upper and lower box correspond to the cases of the $I_{\mu\tau}$ and I_{ee} dominance, respectively. The last column shows the correction to $U_{e3} = s_2$ from exact bimaximal mixing given subdominant $I_{\alpha\beta} \ll I_{\mu\tau}$ or I_{ee}.

consistent with the neutrinoless double beta decay bound as we have $|M_{ee}| = |m_0(c_2^2 \cos 2\theta_3 \pm s_2^2)| \ll |m_0|$.

Let us turn to the solar neutrino mass splitting. As can be seen from Eqs. (13) and (14), the right values for Δm^2_{21} may arise if the mixing anlges satisfy $\cos 2\theta_3 \sim \cos^2 2\theta_1 \sim \Delta m^2_{sol}/\Delta m^2_{atm}$ where the second relation is applied only to the $I_{\mu\tau}$ dominance. However, it appears unnatural to arrange such small values for the tree level mixing angles. It would be more plausible to imagine the situation of exact bimaximal mixing ($\cos 2\theta_1 = \cos 2\theta_3 = 0$) imposed by certain symmetry in tree level mass matrix. Then, the solar neutrino mass splitting could be generated by a smaller loop correction $I_{\alpha\beta}$ other than I_{ee} or $I_{\mu\tau}$. Including now in Eq. (12) this subdominant contribution, one can find the deviations from the leading results, $\Delta m^2_{21} = 0$ and $s_2 = 0$. The result of our calculation is summarized in Table I in the case of the 1-3 degeneracy. We have shown the explicit angle dependences to notify the sign of mass-squared difference which might be distinguishable by the solar neutrino MSW effect. Similar result can be obtained for the 2-3 degeneracy. A few remarks are in order. The desired size of the loop correction for the atmospheric neutrino mass-squared difference $I_{ee,\mu\tau} \approx \Delta m^2_{atm}/m_0^2$. Thus, the degenerate mass $m_0 \sim 1$ eV of cosmological interests needs $I_{ee,\mu\tau} \sim 10^{-3}$ which is a reasonable value for radiative corrections. As can be seen in Table I, the ratio $\Delta m^2_{sol}/\Delta m^2_{atm}$ is roughly given by r or r^2 depending on the flavor structure of the radiative corrections whereas $s_2 \sim r$ for any cases. If the large angle MSW solution to the solar neutrino problem is realized, one needs r or r^2 of the order 10^{-2}, that is, a loop correction $I_{\alpha\beta}$ should be smaller than $I_{ee,\mu\tau}$ by factor of 10^{-2} or 10^{-1}. For the latter case, we get $s_2 \sim 0.1$ which is within the reach of future experiments. We note that a supersymmetric model realizing the case with the I_{ee} dominance and $r = I_{\tau\tau}/I_{ee}$ has been worked out in Ref. [12].

Another possibility is to have the atmospheric neutrino mass splitting given at tree level and the smaller splitting for the solar neutrino mass is driven by loop corrections. This includes almost full degeneracy $|m_1| = |m_2| \simeq |m_3|$ and inverse hierarchy $|m_1| = |m_2| \gg$

$I_{\alpha\beta}$	$I_{12}=0$	$(I_{22}-I_{11})/I_{\alpha\beta}$
I_{ee}	$c_2^2\sin 2\theta_3=0$	$-c_2^2\cos 2\theta_3$
$I_{\mu\mu}$	$\frac{s_2}{c_1^2+s_1^2 s_2^2}=\pm\frac{\sin 2\theta_3}{\sin 2\theta_1}$	$c_1^2\cos 2\theta_3 - s_2\sin 2\theta_1\sin 2\theta_3$
$I_{\tau\tau}$	$\frac{s_2}{s_1^2+c_1^2 s_2^2}=\pm\frac{\sin 2\theta_3}{\sin 2\theta_1}$	$s_1^2\cos 2\theta_3 + s_2\sin 2\theta_1\sin 2\theta_3$
$I_{e\mu}$	$s_2=\cot\theta_1\cot 2\theta_3$	$c_2(+c_1\sin 2\theta_3+s_1 s_2\cos 2\theta_3)$
$I_{e\tau}$	$s_2=-\tan\theta_1\cot 2\theta_3$	$c_2(-s_1\sin 2\theta_3+c_1 s_2\cos 2\theta_3)$
$I_{\mu\tau}$	$\frac{2s_2}{1+s_2^2}=\tan 2\theta_1\tan 2\theta_3$	$-\frac{1}{2}\sin 2\theta_1\cos 2\theta_3 - s_2\cos 2\theta_1\sin 2\theta_3$

TABLE II: The mixing angle relation and the loop contribution to Δm^2_{21} for each dominant $I_{\alpha\beta}$ in the case of the degeneracy, $m_1=m_2\neq m_3$.

$|m_3|$, both of which can be parametrized as

$$\text{II.}\quad (m_1,m_2,m_3)=m_0(1,\pm 1,z)\,,$$

where $z=\pm 1+\delta_a$ with $|\delta_a|=\Delta m^2_{atm}/2m_0^2$ for the almost full degeneracy, or $|z|\ll 1$ with $m_0^2=\Delta m^2_{atm}$ for the inverse hierarchy. We consider the two cases, $m_1=\pm m_2$, separately.

(a) $m_1=m_2=m_0$: As discussed before, the mixing angles satisfy $I_{12}=0$ and the leading contribution to Δm^2_{21} is given by $2m_0^2(I_{22}-I_{11})$. These two quantities are presented in Table II. One can realize that the I_{ee} dominance does not work at all. For $(\alpha\beta)=(\mu\mu),(\tau\tau),(\mu\tau)$, only the *small* solar mixing angle is consistent since the mixing angle relation $s_2\propto\sin 2\theta_3$ has to be put. On the contrary, for $(\alpha\beta)=(e\mu),(e\tau)$, the *large* solar mixing can only be allowed since $s_2\propto\cos 2\theta_3$. Imposing these mixing angle relations, one can see that $I_{22}-I_{11}$ in Table II does not vanish for any (α,β), and thus $\Delta m^2_{sol}\approx m_0^2 I_{\alpha\beta}$. In the case of $|z|\ll 1$, we thus need $I_{\alpha\beta}\approx\Delta m^2_{sol}/\Delta m^2_{atm}\lesssim 10^{-2}$ where the approximate equality is for the large angle MSW solution and may be a little large value for a radiative correction. Here it is worth noting that *e.g.*, for the $I_{\tau\tau}$ dominance, we have $\sin^2 2\theta_3\ll 1$ and

$$\Delta m^2_{21}\simeq 2m_0^2 I_{\tau\tau}s_1^2(c_3^2-s_3^2)\,. \tag{16}$$

For the small solar neutrino mixing to work, we need $s_3^2\ll 1$ for $\Delta m^2_{21}>0$ (or $\Delta m^2_{21}<0$ for $c_3^2\ll 1$), which requires from Eq. (16) that $I_{\tau\tau}>0$. Therefore, $I_{\tau\tau}$ given in Eq. (5) for the supersymmetric standard model does *not* fulfill this condition whereas the usual standard model with $I_{\tau\tau}\approx h_\tau^2\ln(M_X/M_Z)/16\pi^2$ can work.

(b) $m_1=-m_2=m_0$: In this case, no mixing angle relation is imposed. Including the effect of the off-diagonal elements M_{13} and M_{23} generated from loop correction, we find

$$\Delta m^2_{21}=2m_0^2\left[I_{22}-I_{11}+\frac{1}{2}\frac{(z-1)^2}{(z+1)}I_{23}^2+\frac{1}{2}\frac{(z+1)^2}{(z-1)}I_{13}^2\right]\,, \tag{17}$$

where $I_{22}-I_{11}$ is given in Table II for each $I_{\alpha\beta}$. Depending on the mixing angles fixed at tree level, the leading contribution to Δm^2_{sol} may come from $I_{22}-I_{11}$ or the next terms in Eq. (17). That is, we need to have $\Delta m^2_{sol}/\Delta m^2_{atm}\sim I_{\alpha\beta}/\delta_a$ or $I^2_{\alpha\beta}/\delta_a^2$ for $z=\pm 1+\delta_a$, and $\Delta m^2_{sol}/\Delta m^2_{atm}\sim I_{\alpha\beta}$ or $I^2_{\alpha\beta}$ for $|z|\ll 1$. Therefore, the values of Δm^2_{sol} for various solutions to solar neutrino problem can be obtained with appropriate values of $I_{\alpha\beta}$ and δ_a. From Table II, one can see that the leading term $I_{22}-I_{11}\propto I_{\alpha\beta}$ vanishes for the exact bimaximal mixing

with $s_2 = 0$ and $\cos 2\theta_3 = 0$ in the case of $(\alpha\beta) = (ee), (\mu\mu), (\tau\tau)$ and $(\mu\tau)$. Furthermore, for $(\alpha\beta) = (ee)$ and $(\mu\tau)$, I_{13} and I_{23} also vanish and thus no splitting can arise at one-loop level. We would like to stress that either the large or small mixing solution to the solar neutrino problem can be realized as we have no mixing angle relation imposed. For the small mixing solution, the solar neutrino mass-squared difference has further suppression by small mixing angles as $I_{22} - I_{11} \sim I_{\alpha\beta} \sin 2\theta_3$ or $I_{\alpha\beta} s_2$ in the case of $(\alpha\beta) = (e\mu)$ or $(e\tau)$.

In conclusion, we have discussed the general conditions for generating small neutrino mass splittings from the effect of one-loop wave function renormalization of degenerate neutrino masses at tree level. Without assuming any specific model on the structure of lepton flavor violation other than the tree level neutrino mass sector, we identified the flavor dependences of the loop factors which can give rise to the neutrino mass-squared difference and mixings required by the atmospheric and solar neutrino data. For the fully degenerate pattern $|m_1| = |m_2| = |m_3|$, the atmospheric neutrino mass splitting can be obtained when the dominant loop correction comes from I_{ee} or $I_{\mu\tau}$ in the cases of $m_1 = m_3$ and $m_2 = m_3$. In each case, the smaller loop corrections are examined to generate the solar neutrino mass splitting. For all the cases, it turns out that the so-called bimaximal mixing can only be realized. For the partially degenerate case $|m_1| = |m_2| \neq |m_3|$, we identified $I_{\alpha\beta}$ with which the desired mass splitting and the (large or small) mixing angle for the solar neutrino oscillations can be obtained. Our results may be useful for explicit model building along this line.

Acknowledgement: The author is supported by the BK21 program of Ministry of Education.

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81** (1998) 1562.

[2] Homestake Collaboration, B.T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505; Kamiokande Collaboration, K.S. Hirata *et al.*, Phys. Rev. Lett. **77** (1996) 1683; GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. **B388** (1996) 384; SAGE Collaboration, D.N. Abdurashitov *et al.*, Phys. Rev. Lett. **77** (1996) 4708; Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **82** (1999) 2430.

[3] For a recent global analysis, see M.C. Gonzalez-Garcia *et al.*, hep-ph/0009350.

[4] CHOOZ Collaboration, A. Apollonio *et al.*, Phys. Lett. **B420** (1998) 397; Palo Verde experiment, Phys. Rev. Lett. **84** (2000) 309.

[5] L. Baudis *et al.*, hep-ex/9902014.

[6] LSND Collaboration, C. Athanassopoulos, *et.al.*, Phys. Rev. Lett. **75** (1995) 2650; **81** (1998) 1774.

[7] P.H. Chankowski and Z. Pluciennik, Phys. Lett. **B316** (1993) 312; K. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. **B319** (1993) 191; M. Tanimoto, Phys. Lett. **B360** (1995) 41.

[8] J. Ellis and S. Lola, Phys. Lett. **B458** (1999) 310; N. Haba *et al.*, Prog. Theor. Phys. **103** (2000) 367; Eur. Phys. J. **C10** (1999) 177; Eur. Phys. J. **C14** (2000) 347; J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. **B556** (1999) 3; Nucl. Phys. **B569** (2000) 82; JHEP **9909** (1999) 015; Nucl. Phys. **B573** (2000) 652; E. Ma, J. Phys. **G25**, 97 (1999); R. Barbieri, G.G. Ross and A. Strumia, JHEP 9910:020 (1999); P.H. Chankowski, W. Krolikowski and S. Pokorski, Phys. Lett. **B473** (2000) 109; A.S. Dighe and A.S. Joshipura, hep-ph/0010079.

[9] G. Bhattacharyya, hep-ph/9709395; B.C. Allanach, A. Dedes and H.K. Dreiner, Phys. Rev. **D60** (1999) 075014.

[10] For recent works, see *e.g.*, E.J. Chun and S.K. Kang, Phys. Rev. **D61** (2000) 075012; S. Davidson and M. Losada, hep-ph/0010325.

[11] E.J. Chun and S. Pokorski, Phys. Rev. **D62** (2000) 053001.

[12] P.H. Chankowski, A. Ioannisian, S. Pokorski and J.W.F. Valle, hep-ph/0011150.